Maple Assignment Fall 1998 Solution

A motorcyclist starts from rest at \( x = 0 \) and accelerates uniformly along a horizontal track. The times taken by the motorcyclist to pass photogates located at 10 meter intervals are measured. The positions (measured in m) of the photogates are \( x = 0, 10, 20, 30, 40 \) and 50 and, the motorcycle passes these photogates at the times (measured in s) \( t = 0, 1.63, 2.33, 2.83, 3.31 \) and 3.79 respectively.

a) Write down an equation for the distance "\( x \)" to each one of the photogates from the origin in terms of the time "\( t \)" and the constant acceleration "\( a_1 \)". No Maple is needed here.

We expect them to write: \( x = x_0 + (1/2)a_1t^2 \) or something equivalent to this.

b) Using the fit command in Maple as shown in the sample program, find the straight-line fit of the data to the equation in (a) above. What is the acceleration "\( a_1 \)"?

c) Plot the data and the straight-line fit on the same graph of "\( x \)" versus \( t^2 \).

```
> d:={0,10,20,30,40,50}; t:={0,1.63,2.33,2.83,3.31,3.79};
> tt:={0,1.63^2,2.33^2,2.83^2,3.31^2,3.79^2};
> with(stats):
> fit[leastsquare][[d,tt],d=a*tt+b,{a,b}][[0,10,20,30,40,50],[0,2.66,5.43,8.01,10.96,14.36]];

\[ d = 3.517680051 \, tt + 0.7162820493 \]

The acceleration is \( a_1 = 3.52 \times 2 = 7.1 \) m/s\(^2\).

> L:=[[0,0],[2.66,10],[5.43,20],[8.01,30],[10.96,40],[14.36,50]];
> plot([L,3.518*tt+0.716],tt=0..15,style=[point,line]);
```
A second experiment is conducted on the same motorcyclist traveling with a different constant acceleration "a2" and the speeds are measured by detectors mounted on posts numbered \( j = 1, 2, 3, 4, 5, 6 \). The distance between successive posts is \( d_0 = 10 \text{ m} \). The first post (\( j = 1 \)) is located at an unknown position \( x_1 \) from the origin. Note that \( v(j) \) is the speed of the motorcyclist at post number "\( j \)". The equation for \( v_2(j) \) in terms of the speed \( v(1) \) at post 1, is given by

\[
 v_2(j) = v_2(1) + 2a'do(j - 1)
\]

where, \( v_2(1) = 2a'x_1 \).

The speeds (in m/s) are \( v = 14.0, 18.3, 21.7, 24.6, 27.5 \) and 30.0 at posts \( j = 1, 2, 3, 4, 5 \) and 6 respectively.

d) Using the fit command in Maple as shown in the sample program, find the straight-line fit of the data to the equation \( v_2(j) = v_2(1) + 2a_2d_0(j - 1) \) and find the acceleration "\( a' \)" and "\( x_1 \).

```maple
> with(stats):
> fit[leastsquare][vsq,j,vsq=c1+c2*(j-1),{c1,c2}]([[196,334.9,470.9,605.2,756.3,900],[1,2,3,4,5,6]]);

vsq = 52.03333334 + 140.5285714 j
```
So we see that \( c_1 - c_2 = 52.03 \) and \( c_2 = 140.53 \). Thus \( c_1 = 192.6 \) and \( c_2 = 2a'do=140.53 \). Thus \( c_1 = 2a'x_1 = 192.6 \).

```maple
> aprime:=140.53/(2*10);x1:=192.6/(2*aprime);
```

\( aprime := 7.026500000 \)

\( x1 := 13.70525867 \)

```maple
> vsq:=[[1,196],[2,334.9],[3,470.9],[4,605.2],[5,756.3],[6,900]];
> plot([vsq,140.5285714*j+52.033333334],j=1..6,style=[point,line]);
```

\( vsq := [[1,196],[2,334.9],[3,470.9],[4,605.2],[5,756.3],[6,900]] \)