MATLAB exercises

1. Use rand function to generate 100 random numbers. Treating the numbers less than or equal to 0.5 as zero and the other half as 1, simulate the experiment of a coin toss (1-Heads) and 0-Tails). Starting from 10, increase the numbers in steps of ten and establish how many numbers one needs to reach the case of an unbiased coin. Is it possible to understand the concept of mutually exclusive events from this experiment?

2. How will you prove using a set of random numbers, the probabilities associated with the roll of a fair die?

3. Use two sets of random numbers to establish the probability results relating to the toss of two fair coins simultaneously. Can you also establish the results on independence of two events?

4. Generate three sets of random numbers. For the first set, create zeros and ones as described in Problem (1). For the second set, generate uniform random numbers in the range {0,2} and take zero to be all the numbers less than or equal 0.5 and the rest to be 1. For the third set, generate uniform random numbers in the range {0,3} take zeros to come from all numbers less than or equal to 0.5 and the rest to be 1. Can you explain what the probabilities of zero’s and 1’s in the three sets mean? Can you establish Bayes’ rule of total probability from this experiment?

5. Generate two groups of 1000 uniform random numbers (x_i and y_i, i=1, 2, .. 1000). Plot histograms of z_i = x_i + y_i and w_i = x_i - y_i. Also calculate the mean and variance of z and w. Comment on your results.

6. Generate two groups of 100 uniform random numbers (x_i and y_i, i=1, 2, ..100). Now compute

\[ z_i = \sqrt{-2 \ln(x_i) \cos(2y_i \pi - 1)} \]

Plot a histogram of z_i and compare it to the normal pdf (Hint: use histfit)

7. Generate 200 uniform random numbers (x_i, i=1,2,..200). Calculate \( u_i = -\ln(x_i) \) and plot its histogram.

8. Generate two groups of 200 uniform random numbers (x_i and y_i, i=1, 2, . .200). Now compute

\[ z_i = \sqrt{-2 \ln(x_i) \cos(2y_i \pi - 1)} \quad w_i = \sqrt{-2 \ln(x_i) \sin(2y_i \pi - 1)} \]

Compare the histograms in z and w. Comment on your results.
9. Generate Uniformly distributed random numbers. Plot the histogram and cumulative distribution function. Compare the results to theoretical plots. Use 10 different sets of random numbers and using Central Limit Theorem, generate Gaussian Random Numbers. Generate Gaussian directly using MATLAB and compare the results.

10. Generate a Gaussian random number set. Plot the histogram. Plot the distribution function. Compare the plots to corresponding theoretical plots. Start with a set of 100 and increase it 1000 in steps of 200. Comment on your results.

11. Now use two Gaussian random number sets and obtain the Rayleigh and exponential from the Gaussian. Conversely, generate a Gaussian random number from a Rayleigh distributed random number set and Uniformly distributed random number set.

12. Generate a set of Poisson distributed random numbers (means ranging from 1-10). Increase the mean. Observe how the histogram changes. For example, generate a Poisson number set with a mean of 25. Generate a Gaussian set with a mean of 25 and standard deviation of 5. Compare this Gaussian random number set to the Poisson with mean equal to 25. Discuss your observation.

13. Generate two sets of Gaussian random numbers, one with a mean of zero and standard deviation of 1 and another one with a mean of 1 and standard deviation of 1. Plot the histograms on the same graph.

   Now for the first set, find the probability that the numbers are larger than 0.5, by counting the numbers that are larger than 0.5.

   For the second set, find the probability that the numbers are less than 0.5 similarly. Explain your results. What do these results mean? Where are they likely to be used? (Concept of ROC)

14. Generate a set of binomial random numbers. Demonstrate the relationship between Binomial and normal distributions.

Problems/Exercises

1. In a digital communication system, messages are encoded into 0's and 1's. Because of the noise in the system, incorrect symbol is often received. Assume that the probability of a 0 being transmitted is 0.4 and the probability of 1 being transmitted is 0.6. If you assume that the probability of a 0 being received as a 1 is 0.08 and the probability of 1 being received as a 0 is 0.05, find (i) the probability that a received zero was transmitted as a 0, (ii) probability that a received 1 was transmitted as a 1 and (iii) the probability that any symbol is being received in error.

2. In a ternary communication system, a 3 is transmitted three times more frequently than a 1 and a 2 is transmitted two times more frequently than a 1. Probability of receiving a 2
or 3 when a 1 has been transmitted is $A/2$. Corresponding value for a 2 is $B/2$ and for a 3, it is $C/2$. At the receiver a 1 is observed. What is the probability that a 1 was transmitted?

3. In a multiple choice examination, the fraction of students who know the answers is 20%. They write the correct answers in the test. The rest of the students guess the answers. If there are 30 questions in the test, the probability of getting the right answers by guessing is $1/30$. If a student got all of them right, what is the probability that she/he new all the answers.

4. A typist sometimes makes mistakes by hitting a key right or left of the intended key with a probability of 0.02. The letters E, R, and T are adjacent to one another on a standard QWERTY keyboard. Note that in English language, typically E, R, and T occur with probabilities of 0.1031, 0.0484 and 0.0796 respectively.

(a) What is the probability that R appears in the text typed by this typist?
(b) What is the probability that the letter R appearing is in error?

5. Is the following diagnostic test (for cancer) being used in a hospital good?

$A$: the event that the cancer test states that a person has cancer
$B$: the event that any person has cancer (Occurrence of cancer among population)
$A^c$: the event that the test states that the person has no cancer
$B^c$: the event that a person has no cancer.

Also given that $p(B^c)=0.004$; $p(A/B)=p(A^c/B^c)=0.96$

6. You are dealt three cards. What is the probability that all will be of the same suit?
7. A box contains 3 damaged and 8 good books. You select two books at random. What is the probability that at least one is damaged?

8. Numbers 1 thro' 6 are given. Find the number of ways of picking two numbers (a) without replacement where order does not matter, (b) without replacement where order does matter (c) repetition is allowed where order does matter, and (d) repetition is allowed where order does not matter.

9. A coin is tossed 5 times. How many ways can one get 3 heads?

10. (a) In how many ways can 8 people be seated on a bench?  
    (b) In how many ways can 8 people be seated around a circular table?

11. How many ways are there to form a committee of six students from 14 male (M) and 1 female (F) students if (a) F must always be on the Committee, (b) F must not be on the Committee, and (c) there are no restrictions.

12. In a quiz containing five true/false questions, what is the probability that the student will pass (i.e.; get 60% or more) if he/she picks answers randomly? (Use Bernoulli Theorem)\( p(\text{true})=p(\text{false})=0.5 \)

13. Consider a binary code with 4 bits (0 or 1). What is the probability of seeing two 1’s? What is the probability of seeing three 1’s?

14. A person owns two PCs, A & B, and has trouble starting them. The probability that both will start is 0.1; the probability that the B starts and the A does not start is 0.2; probability that neither starts is 0.4.

(a) Find the probability that A will start.
(b) Find the probability that A will start, given that the B will start.
(c) Find the probability that PC B will start, given that A does not start.

15. What are the values of the constants (all positive) so that the following functions are probability density functions?

\[
 f_x(x) = Ae^{-\alpha x} U(x) \\
 f_x(x) = Be^{\beta x} U(-x) \\
 f_x(x) = Ce^{-\gamma |x|} \\
 f_x(x) = \frac{D}{x^2 + x^2} 
\]

16. Show that \( E\{X^2\} \geq [E\{X\}]^2 \) for any random variable, X.
17. Let \( f(x) \) be the density function and \( F(x) \) be the distribution function. Which of the following statements are TRUE.

- \( f(x) \) cannot be larger than 1.
- \( F(x) \) cannot be larger than 1.
- \( f(x) \) cannot decrease with \( x \).
- \( F(x) \) cannot decrease with \( x \).
- \( f(x) \) cannot be negative.
- \( F(x) \) cannot be negative.
- \( f(x) \) can be infinite.
- Area under \( f(x) \) must be 1.
- Area under \( F(x) \) must be 1.

18. Telephone calls are known to last an average of 2 minutes. Assuming exponential distribution for the call duration,

(a) Find the probability that the call will last less than 2 minutes.
(b) Find the probability that the call lasts longer than the average.
(c) Suppose the call has already lasted 4 minutes. Find the probability that it will last less than 2 more minutes.
(d) Suppose a call lasted two minutes already. What is the probability that it will last longer than the average duration?

19. Choose a number \( X \) at random from interval \( \{0,2\} \). What is the probability that the first digit to the right of the decimal point is a 4? That the second digit is a 4?

20. A Gaussian random variable has a mean of 2 and standard deviation of 2. Find \( P\{X>1.0\} \) and \( P\{X \leq -1\} \).

21. Electronic noise voltage \( V \) is modeled in terms of a Gaussian distribution with a mean of 0.0 volts and a variance of 16 volts-square. (a) Find the probability that the absolute values of the noise voltage will be less than 1.0 volt? (b) For what value of the voltage, \( v \), will the probability, \( \text{Prob}\{|V| \leq v\} \) be equal to 0.95? (c) What is the probability that the measured noise voltage will be equal to 0.9 volts? (d) What is the probability that the noise voltage will lie between 4 volts and 8 volts?
22. Height of the clouds above ground is a Gaussian with a mean of 1500 meters and a standard deviation of 500 meters. What is the probability that the clouds will be higher than 2750 meters? What is the probability that clouds will be at a height of 2000 meters?

23. A production line manufactures 1000 W resistors that must satisfy 10% tolerance. If the resistance is adequately described by a Gaussian random variable with a mean of 1000 W and standard deviation of 40 W, what fraction of the resistors are likely to be rejected?

24. A random variable $X$ is Gaussian with a mean of 0 and standard deviation of 1. What is the probability that $|X| > 2$? What is the probability that $X > 2$? What is the probability that $X = 0$?

25. The lifetime of a filament lamp in hours, $T$, is given by an exponential distribution $f_T(t) = \frac{1}{100} \exp\left(-\frac{t}{100}\right) U(t)$.

(a) What is the average lifetime?

(b) What is the probability that the filament breaks in less than two hours?

(c) What is the probability that the lifetime exceeds 100 hours?

(d) Given that the bulb lasts beyond 200 hours, what is the probability that it will last beyond 300 hours?

26. Tommy is to arrive at 7:00 P.M to meet Tammy. Because of random traffic pattern, the actual arrival time is uniformly distributed between 7:00 and 7:30 P.M. Tammy arrives to meet Tommy at 7:15 P.M. What is the probability that (a) Tammy arrived before Tommy (b) Tammy waits less than 5 minutes, and (c) Tammy waits less than 10 minutes, given that she arrived before Tommy.

27. The number of cars that arrive at a toll booth during any ten minute period is Poisson distributed, $p_X(x) = f_X(x) = \sum_{k=0}^{x} e^{-\lambda} \frac{\lambda^k}{k!} \delta(x-k)$, find the probability that more than 4 cars arrive at the window during any ten minute period.

28. The lifetime of a component expressed in weeks is a Rayleigh distributed random variable as follows: $f_X(x) = \frac{x}{100} e^{-\frac{x^2}{200}} U(x)$

Given that the system will survive beyond 10 weeks, what is the probability that it will survive beyond 15 weeks?

29. The envelope of the output signal of a radar system that is receiving only noise (NO SIGNAL) is Rayleigh distributed as follows:
\[ f_X(x) = xe^{-\frac{x^2}{2}} U(x) \]

The radar gets a false target detection of the signal exceeds a threshold voltage of \( X_T \). How large must \( X_T \) be so that the probability of false alarm is 0.0011?

30. The density function of \( X \) is \( f(x) = e^{-2|x|} \). What is the distribution function? What is the probability that \( \{X^2 \geq 2\} \)?

31. A communication satellite is designed to have a mean life time of 6 years. If the actual time of failure is an exponentially distributed random variable, find
   (a) the probability that the satellite will fail sooner than five years.
   (b) the probability that the satellite will survive beyond 10 years
   (c) the probability that the satellite will fail during the sixth year.

32. The resistance \( Y \) of a structural element and the force \( X \) applied to it are random variables described by \( f_{X,Y}(x,y) = abe^{-(ax+by)} U(x)U(y) \). If the failure of the structure is characterized by \( \{Y \leq X\} \), calculate the probability of failure.

33. On a dartboard, the location of the dart (w. r. t. the bull’s-eye) can be described by a circular Gaussian \( \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}} \). If for a good marksman, 90% of the darts fall within a circle of diameter 6 cm, what is the value of \( \sigma^2 \).